



**A Selection of Mathematical Problems Tackled this Year so far...
(Ordered Roughly by Increasing Level of Difficulty)**

What is the sum of 25% of 2018 and 2018% of 25? (Int.Kang.Pink2018Q7)

The sum of five consecutive integers is 10^{2018} . What is the middle number? (Int.Kang.Pink2018Q3)

A particular integer is the smallest multiple of 72, each of whose digits is either 0 or 1. How many digits does this integer have? (Int.Challenge2018Q20)

In a particular group of people, some always tell the truth, the rest always lie. There are 2016 in the group. One day, the group is sitting in a circle. Each person in the group says, "Both the person on my left and the person on my right are liars." What is the difference between the largest and smallest number of people who could be telling the truth? (Int.Challenge2016Q22)

A list of 5 positive integers has mean 5, mode 5, median 5 and range 5. Identify all possible such lists. (Int.Challenge2018Q24)

How many nine-digit integers of the form 'pqrpqrpqr' are multiples of 24?
(Note that p, q and r need not be different.)
(Int.OlympiadHamilton2018Q2)

Two real numbers x and y satisfy the equation $x^2 + y^2 + 3xy = 2015$. What is the maximum possible value of xy? (Int.OlympiadMaclaurin2015Q2)

In 1984 the engineer and prolific prime-finder Harvey Dubner found the biggest known prime each of whose digits is either a one or a zero. The prime can be expressed as $[(10^{641} \times (10^{640} - 1)) / 9] + 1$. How many digits does this prime have? (Int.Challenge2013Q25)

The sequence of functions $F_1(x), F_2(x), \dots$ satisfies the following conditions:

$$F_1(x) = x, \quad \text{and} \quad F_{n+1}(x) = 1 / (1 - F_n(x))$$

The integer C is a three-digit cube such that $F_C(C) = C$. What is the largest possible value of C? (SeniorKangaroo2018Q19)

A set of 12 coins are all identical in terms of their appearance. One of the coins, however, is slightly heavier or lighter (and you do not know which) than the remaining coins, all 11 of which are of the same weight. Using a simple set of balancing scales, **and with ONLY three weighings**, how can you determine which coin is the odd one out, and whether it is heavier or lighter? (A well-known classic)

Positive integers p, a and b satisfy the equation $p^2 + a^2 = b^2$. Prove that if p is a prime greater than 3, then a is a multiple of 12 and $2(p + a + 1)$ is a perfect square. (BMO1 2014Q2)

Let ABC be a triangle. Let S be the circle through B tangent to CA at A and let T be the circle through C tangent to AB at A. The circles S and T intersect at A and D. Let E be the point where the line AD meets the circle ABC. Prove that D is the midpoint of AE. (BMO1 2012Q6)

Let a and b be positive integers such that $ab + 1$ divides $a^2 + b^2$. Show that $(a^2 + b^2) / (ab + 1)$ is the square of an integer. (IMO 1988Q6. EXTREMELY CHALLENGING)